



## SEPARATED AND DISCONNECTED SETS IN BIMINIMAL STRUCTURE SPACES

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### Abstract

The purpose of this paper is to introduce the concept of separated sets, connected and disconnected sets in biminimal structure spaces. We obtain some fundamental properties of their sets. Moreover, we define connected and disconnected spaces and some properties of their spaces are obtained.

### 1. Introduction

The minimal structure spaces were introduced by Popa and Noiri [6] in

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2000 and they also introduced the concepts of  $m_X$ -open set and  $m_X$ -closed set. In [5], Noiri and Popa defined  $m_X$ -compactness and  $m_X$ -connectedness and investigated their properties. Bitopological Spaces were introduced by Kelly [2], and in 2010, Boonpok [1] introduced the notion of biminimal structure spaces, biminimal structure subspaces and study some fundamental properties of  $m_X^1 m_X^2$ -closed sets and  $m_X^1 m_X^2$ -open sets. In this paper, we introduce the concept of separated sets, connected and disconnected sets and study some fundamental properties of their sets. Moreover, we define connected and disconnected spaces and some properties of their spaces are obtained.

## 2. Preliminaries

In this section, we recall some notions, notations and previous results.

**Definition 2.1** [5]. A subfamily  $m_X$  of the power set  $P(X)$  of a nonempty set  $X$  is called a *minimal structure* (briefly *m-structure*) on  $X$  if  $\emptyset \in m_X$  and  $X \in m_X$ . Each member of  $m_X$  is said to be  $m_X$ -open and the complement of an  $m_X$ -open set is said to be  $m_X$ -closed.

**Definition 2.2** [3]. Let  $X$  be a nonempty set and  $m_X$  be an *m-structure* on  $X$ . For a subset  $A$  of  $X$ , the  $m_X$ -closure of  $A$  and  $m_X$ -interior of  $A$  are defined as follows:

1.  $m_X Cl(A) = \bigcap \{F : A \subseteq F, X \setminus F \in m_X\}$ ,
2.  $m_X Int(A) = \bigcup \{U : U \subseteq A, U \in m_X\}$ .

**Lemma 2.3** [3]. Let  $X$  be a nonempty set and  $m_X$  be an *m-structure* on  $X$ . For subsets  $A$  and  $B$  of  $X$ , the following hold:

1.  $m_X Cl(X \setminus A) = X \setminus m_X Int(A)$  and  $m_X Int(X \setminus A) = X \setminus m_X Cl(A)$ ,
2. If  $(X \setminus A) \in m_X$ , then  $m_X Cl(A) = A$  and if  $A \in m_X$ , then  $m_X Int(A) = A$ ;

3.  $m_X Cl(\emptyset) = \emptyset$ ,  $m_X Cl(X) = X$ ,  $m_X Int(\emptyset) = \emptyset$  and  $m_X Int(X) = X$ ,

4. If  $A \subseteq B$ , then  $m_X Cl(A) \subseteq m_X Cl(B)$  and  $m_X Int(A) \subseteq m_X Int(B)$ ,

5.  $A \subseteq m_X Cl(A)$  and  $m_X Int(A) \subseteq A$ ,

6.  $m_X Cl(m_X Cl(A)) = m_X Cl(A)$  and  $m_X Int(m_X Int(A)) = m_X Int(A)$ .

**Lemma 2.4** [3]. Let  $X$  be a nonempty set with a minimal structure and  $A$  be a subset of  $X$ . Then  $x \in m_X Cl(A)$  if and only if  $U \cap A \neq \emptyset$  for every  $U \in m_X$  containing  $x$ .

**Definition 2.5** [3]. A minimal structure  $m_X$  on a nonempty set  $X$  is said to have property  $\mathbb{B}$ , if the union of any family of subsets belonging to  $m_X$  belongs to  $m_X$ .

**Lemma 2.6** [5]. For a minimal structure  $m_X$  on a nonempty set  $X$ , the following are equivalent:

1.  $m_X$  has property  $\mathbb{B}$ ,
2. If  $m_X Int(V) = V$ , then  $V \in m_X$ ,
3. If  $m_X Cl(F) = F$ , then  $X \setminus F \in m_X$ .

**Lemma 2.7** [5]. Let  $X$  be a nonempty set and  $m_X$  be a minimal structure on  $X$  satisfying property  $\mathbb{B}$ . For a subset  $A$  of  $X$ , the following properties hold:

1.  $A \in m_X$  if and only if  $m_X Int(A) = A$ ,
2.  $A$  is  $m_X$ -closed if and only if  $m_X Cl(A) = A$ ,
3.  $m_X Int(A) \in m_X$  and  $m_X Cl(A)$  is  $m_X$ -closed.

**Definition 2.8** [1]. Let  $X$  be a nonempty set and  $m_X^1, m_X^2$  be minimal

structures on  $X$ . A triple  $(X, m_X^1, m_X^2)$  is called a *biminimal structure space* (briefly *bim-space*).

**Definition 2.9** [1]. A subset of a biminimal structure space  $(X, m_X^1, m_X^2)$  is called  $m_X^1 m_X^2$ -closed if  $A = m_X^1 Cl(m_X^2 Cl(A))$ . The complement of  $m_X^1 m_X^2$ -closed set is called  $m_X^1 m_X^2$ -open.

**Proposition 2.10** [1]. Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space. If  $A$  and  $B$  are  $m_X^1 m_X^2$ -closed subsets of  $(X, m_X^1, m_X^2)$ , then  $A \cap B$  is  $m_X^1 m_X^2$ -closed.

**Proposition 2.11** [1]. Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space. Then  $A$  is  $m_X^1 m_X^2$ -open subset of  $(X, m_X^1, m_X^2)$  if and only if  $A = m_X^1 Int(m_X^2 Int(A))$ .

**Proposition 2.12** [1]. Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space. If  $A$  and  $B$  are  $m_X^1 m_X^2$ -open subsets of  $(X, m_X^1, m_X^2)$ , then  $A \cup B$  is  $m_X^1 m_X^2$ -open.

**Definition 2.13** [1]. Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $Y$  be a subset of  $X$ . Define minimal structures  $m_Y^1$  and  $m_Y^2$  as follows:  $m_Y^1 = \{A \cap Y : A \in m_X^1\}$  and  $m_Y^2 = \{B \cap Y : B \in m_X^2\}$ . A triple  $(Y, m_Y^1, m_Y^2)$  is called a *biminimal structure subspace* (briefly *bim-subspace*) of  $(X, m_X^1, m_X^2)$ .

Let  $(Y, m_Y^1, m_Y^2)$  be a biminimal structure subspace of  $(X, m_X^1, m_X^2)$ , and let  $A$  be a subset of  $Y$ . The  $m_Y$ -closure and  $m_Y$ -interior of  $A$  with respect to  $m_Y^i$  are denoted by  $m_Y^i Cl(A)$  and  $m_Y^i Int(A)$ , respectively (for  $i = 1, 2$ ). Then  $m_Y^1 Cl(A) = Y \cap m_X^1 Cl(A)$  and  $m_Y^2 Cl(A) = Y \cap m_X^2 Cl(A)$ .

**Proposition 2.14** [1]. Let  $(Y, m_Y^1, m_Y^2)$  be a biminimal structure subspace of  $(X, m_X^1, m_X^2)$  and  $F$  be a subset of  $Y$ . If  $F$  is  $m_X^1 m_X^2$ -closed, then  $F$  is  $m_Y^1 m_Y^2$ -closed.

### 3. Main Results

In this section, we define the new definitions and construct their properties of separated and disconnected sets in biminimal structure spaces.

**Lemma 3.1.** Let  $(X, m_X)$  be a minimal structure space and  $G \subseteq X$ . If  $G$  is  $m_X$ -open, then  $G \cap A = \emptyset$  if and only if  $G \cap m_X Cl(A) = \emptyset$ .

**Proof.** Let  $(X, m_X)$  be a minimal structure space and  $G \subseteq X$ . Suppose that  $G$  is  $m_X$ -open.

( $\Rightarrow$ ) Assume that  $G \cap A = \emptyset$ . Suppose that  $G \cap m_X Cl(A) \neq \emptyset$ . Thus, there exist  $x \in G$  and  $x \in m_X Cl(A)$ . By Lemma 2.4, we have  $U \cap A \neq \emptyset$  for every  $U \in m_X$  containing  $x$ . Since  $G$  is  $m_X$ -open and  $x \in G$ ,  $G \cap A \neq \emptyset$ . Contradiction with  $G \cap A = \emptyset$ . Thus,  $G \cap m_X Cl(A) = \emptyset$ .

( $\Leftarrow$ ) Assume that  $G \cap m_X Cl(A) = \emptyset$ . Since  $A \subseteq m_X Cl(A)$ , therefore  $G \cap A = \emptyset$ .

**Definition 3.2.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and let  $A$  and  $B$  be two nonempty subsets of  $X$ . Then  $A$  and  $B$  are called  $m_X$ -separated sets if and only if  $m_X^i Cl(A) \cap B = \emptyset$  and  $A \cap m_X^i Cl(B) = \emptyset$ , where  $i = 1, 2$ .

**Example 3.3.** Let  $X = \{1, 2, 3\}$ . Define  $m$ -structures  $m_X^1$  and  $m_X^2$  on  $X$  as follows:  $m_X^1 = \{\emptyset, \{1, 3\}, \{2, 3\}, X\}$  and  $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$ . Then  $\{1\}$  and  $\{2\}$  are  $m_X$ -separated sets. But  $\{1\}$  and  $\{2, 3\}$  are not  $m_X$ -separated sets.

**Theorem 3.4.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space,  $A$  and  $B$  be two nonempty  $m_X^i$ -open subsets of  $X$ , where  $i = 1, 2$ . Then  $A$  and  $B$  are  $m_X$ -separated sets if and only if  $A \cap B = \emptyset$ .

**Proof.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space,  $A$  and  $B$  be a nonempty  $m_X^i$ -open subset of  $X$ , where  $i = 1, 2$ .

( $\Rightarrow$ ) Assume that  $A$  and  $B$  are  $m_X$ -separated sets. Thus,  $m_X^i Cl(A) \cap B = \emptyset$  and  $A \cap m_X^i Cl(B) = \emptyset$ , where  $i = 1, 2$ . Hence, we have  $A \cap B = \emptyset$ .

( $\Leftarrow$ ) Assume that  $A \cap B = \emptyset$ . Since  $A$  and  $B$  are  $m_X^i$ -open subsets of  $X$  and Lemma 3.1, we have  $m_X^i Cl(A) \cap B = \emptyset$  and  $A \cap m_X^i Cl(B) = \emptyset$ , where  $i = 1, 2$ . Thus,  $A$  and  $B$  are  $m_X$ -separated sets.

**Theorem 3.5.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space,  $A$  and  $B$  be two nonempty subsets of  $X$ ,  $A \cap B = \emptyset$  and  $m_{A \cup B}^i$  satisfying property  $\mathbb{B}$ . Then  $A$  and  $B$  are  $m_X$ -separated sets if and only if  $A$  and  $B$  are  $m_{A \cup B}^i$ -open and  $m_{A \cup B}^i$ -closed, where  $i = 1, 2$ .

**Proof.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space,  $A$  and  $B$  be two nonempty subsets of  $X$ ,  $A \cap B = \emptyset$  and  $m_{A \cup B}^i$  satisfying property  $\mathbb{B}$ .

( $\Rightarrow$ ) Assume that  $A$  and  $B$  are  $m_X$ -separated sets. Since  $A$  and  $B$  are  $m_X$ -separated sets,  $m_X^i Cl(A) \cap B = \emptyset$  and  $A \cap m_X^i Cl(B) = \emptyset$ , where  $i = 1, 2$ . Let  $Y = A \cup B$ . Then

$$\begin{aligned} m_Y^i Cl(A) &= m_X^i Cl(A) \cap Y = m_X^i Cl(A) \cap (A \cup B) \\ &= A \cup (m_X^i Cl(A) \cap B) \\ &= A \cup \emptyset = A. \end{aligned}$$

Thus by Lemma 2.7(2),  $A$  is  $m_{A \cup B}^i$ -closed. Similarly, we have  $B$  is  $m_{A \cup B}^i$ -closed, where  $i = 1, 2$ . Since  $A \cap B = \emptyset$ ,  $A = (A \cup B) \setminus B$  and  $B = (A \cup B) \setminus A$ . Hence,  $A$  and  $B$  are  $m_{A \cup B}^i$ -open, where  $i = 1, 2$ .

( $\Leftarrow$ ) Let  $Y = A \cup B$ . Since  $A$  and  $B$  are  $m_{A \cup B}^i$ -open, where  $i = 1, 2$ , and  $A \cap B = \emptyset$ . By Lemma 3.1, we have  $m_Y^i Cl(A) \cap B = \emptyset$  and  $A \cap m_Y^i Cl(B) = \emptyset$ , where  $i = 1, 2$ . It follows that

$$\begin{aligned} m_X^i Cl(A) \cap B &= m_X^i Cl(A) \cap (Y \cap B) = (m_X^i Cl(A) \cap Y) \cap B \\ &= m_Y^i Cl(A) \cap B = \emptyset, \end{aligned}$$

and similarly we have,  $A \cap m_X^i Cl(B) = \emptyset$ . Therefore,  $A$  and  $B$  are  $m_X$ -separated sets.

**Theorem 3.6.** Let  $(Y, m_Y^1, m_Y^2)$  be a biminimal structure subspace of  $(X, m_X^1, m_X^2)$  and  $A, B \subseteq Y$ . Then  $A$  and  $B$  are  $m_X$ -separated sets if and only if  $A$  and  $B$  are  $m_Y$ -separated sets.

**Proof.** Let  $(Y, m_Y^1, m_Y^2)$  be a biminimal structure subspace of  $(X, m_X^1, m_X^2)$  and  $A, B \subseteq Y$ .

( $\Rightarrow$ ) Assume that  $A$  and  $B$  are  $m_X$ -separated sets.

Since  $A$  and  $B$  are  $m_X$ -separated sets,  $m_X^i Cl(A) \cap B = \emptyset$  and  $A \cap m_X^i Cl(B) = \emptyset$ , where  $i = 1, 2$ . Therefore

$$m_Y^i Cl(A) \cap B = m_X^i Cl(A) \cap B \cap Y = \emptyset$$

and

$$A \cap m_Y^i Cl(B) = A \cap m_X^i Cl(B) \cap Y = \emptyset,$$

where  $i = 1, 2$ . Hence,  $A$  and  $B$  are  $m_Y$ -separated sets.

( $\Leftarrow$ ) Suppose that  $A$  and  $B$  are  $m_Y$ -separated sets. By assumption and  $A, B \subseteq Y$ , thus

$$m_X^i Cl(A) \cap B = m_X^i Cl(A) \cap Y \cap B = m_Y^i Cl(A) \cap B = \emptyset$$

and

$$A \cap m_X^i Cl(B) = A \cap Y \cap m_X^i Cl(B) = A \cap m_Y^i Cl(B) = \emptyset,$$

where  $i = 1, 2$ . Hence,  $A$  and  $B$  are  $m_X$ -separated sets.

**Definition 3.7.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $A \subseteq X$ . We call  $A$  is  $m_X$ -disconnected set if and only if  $A = C \cup D$ , where  $C$  and  $D$  are  $m_X$ -separated sets and we call  $A$  is  $m_X$ -connected set if and only if  $A$  is not  $m_X$ -disconnected set.

A biminimal structure space  $(X, m_X^1, m_X^2)$  is called  $m_X$ -disconnected space if  $X$  is  $m_X$ -disconnected set and  $(X, m_X^1, m_X^2)$  is called  $m_X$ -connected space if  $X$  is  $m_X$ -connected set.

**Example 3.8.** Let  $X = \{1, 2, 3\}$ . Define  $m$ -structures  $m_X^1$  and  $m_X^2$  on  $X$  as follows:  $m_X^1 = \{\emptyset, \{1, 3\}, \{2, 3\}, X\}$  and  $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$ . Since  $\{1\}$  and  $\{2\}$  are  $m_X$ -separated sets,  $\{1, 2\}$  is  $m_X$ -disconnected set. And we have  $X$  is  $m_X$ -connected set. Therefore,  $(X, m_X^1, m_X^2)$  is  $m_X$ -connected space.

**Theorem 3.9.** Let  $(Y, m_Y^1, m_Y^2)$  be a biminimal structure subspace of  $(X, m_X^1, m_X^2)$  and  $A \subseteq Y$ . Then  $A$  is  $m_X$ -disconnected set if and only if  $A$  is  $m_Y$ -disconnected set.

**Proof.** By Theorem 3.6, we have if  $C, D \subseteq Y$ , then  $C$  and  $D$  are  $m_X$ -separated sets if and only if  $C$  and  $D$  are  $m_Y$ -separated sets. Thus,



$A = C \cup D$  is  $m_X$ -disconnected set if and only if  $A = C \cup D$  is  $m_Y$ -disconnected set.

By Theorem 3.9, we have the following corollary.

**Corollary 3.10.** *Let  $(Y, m_Y^1, m_Y^2)$  be a biminimal structure subspace of  $(X, m_X^1, m_X^2)$  and  $A \subseteq Y$ . Then  $A$  is  $m_X$ -connected set if and only if  $A$  is  $m_Y$ -connected set.*

**Theorem 3.11.** *Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $m_X^1, m_X^2$  have property  $\mathbb{B}$ . Thus,  $X$  is  $m_X$ -disconnected set if and only if there exists a nonempty proper subset  $A$  of  $X$  such that  $A$  is  $m_X^i$ -open set and  $m_X^i$ -closed set, where  $i = 1, 2$ .*

**Proof.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $m_X^1, m_X^2$  have property  $\mathbb{B}$ .

( $\Rightarrow$ ) Assume that  $X$  is  $m_X$ -disconnected set. Thus, there exist nonempty sets  $C$  and  $D$  such that  $X = C \cup D$ ,  $C$  and  $D$  are  $m_X$ -separated sets. Since  $C \subseteq m_X^i Cl(C)$  and  $m_X^i Cl(C) \cap D = \emptyset$ ,  $C \cap D = \emptyset$ . It follows that  $C = X \setminus D$  and  $D = X \setminus C$ . From  $X = C \cup D \subseteq C \cup m_X^i Cl(D) \subseteq X$ . We have  $X = C \cup m_X^i Cl(D)$ . Since

$$C \cap m_X^i Cl(D) = \emptyset, C = X \setminus m_X^i Cl(D) = m_X^i Int(X \setminus D) = m_X^i Int(C).$$

By Lemma 2.7, we have  $C$  is  $m_X^i$ -open set. Then we also have  $D$  is  $m_X^i$ -closed set. Since  $D \cap m_X^i Cl(C) = \emptyset$ ,  $D = m_X^i Int(D)$  and Lemma 2.7, we have  $D$  is  $m_X^i$ -open set. Then we also have  $C$  is  $m_X^i$ -closed set, where  $i = 1, 2$ .

( $\Leftarrow$ ) Suppose that there exists a nonempty proper subset  $A$  of  $X$  such that  $A$  is  $m_X^i$ -open set and  $m_X^i$ -closed set, where  $i = 1, 2$ . Let  $B = X \setminus A$ . Thus,

$B$  is a nonempty proper subset of  $X$  and by Lemma 2.7, we have  $B$  is  $m_X^i$ -closed set and  $m_X^i$ -open set, where  $i = 1, 2$ . Therefore,  $X = A \cup B$  and  $A \cap B = \emptyset$ . Since  $A$  and  $B$  are  $m_X^i$ -open set and Lemma 3.1,  $A \cap m_X^i Cl(B) = \emptyset$  and  $m_X^i Cl(A) \cap B = \emptyset$ , where  $i = 1, 2$ . Hence,  $X$  is  $m_X$ -disconnected set.

**Theorem 3.12.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $m_X^1, m_X^2$  satisfy property  $\mathbb{B}$ . Thus,  $X$  is  $m_X$ -disconnected set if and only if  $X = U \cup V$ , where  $U$  and  $V$  are nonempty  $m_X^i$ -open sets,  $i = 1, 2$  and  $U \cap V = \emptyset$ .

**Proof.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $m_X^1, m_X^2$  satisfy property  $\mathbb{B}$ .

( $\Rightarrow$ ) Assume that  $X$  is  $m_X$ -disconnected set. By Theorem 3.11, there exists a nonempty proper subset  $A$  of  $X$  such that  $A$  is  $m_X^i$ -open set and  $m_X^i$ -closed set, where  $i = 1, 2$ . Then by Lemma 2.7,  $X \setminus A$  is  $m_X^i$ -closed set and  $m_X^i$ -open set such that  $X \setminus A$  is a nonempty proper subset of  $X$ . Put  $U = A$  and  $V = X \setminus A$ . Then  $X = U \cup V$ ,  $U \cap V = \emptyset$  and  $U$  and  $V$  are nonempty  $m_X^i$ -open sets,  $i = 1, 2$ .

( $\Leftarrow$ ) Assume that  $X = U \cup V$ , where  $U$  and  $V$  are nonempty  $m_X^i$ -open sets,  $i = 1, 2$  and  $U \cap V = \emptyset$ . Since  $U$  and  $V$  are  $m_X^i$ -open sets and  $U \cap V = \emptyset$ , by Lemma 3.1,  $U \cap m_X^i Cl(V) = \emptyset$  and  $m_X^i Cl(U) \cap V = \emptyset$ , where  $i = 1, 2$ . Therefore,  $X$  is  $m_X$ -disconnected set.

**Theorem 3.13.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $E \subseteq X$  be  $m_X$ -connected set. If  $E \subseteq A \cup B$ ,  $A$  and  $B$  are  $m_X$ -separated set, then  $E \subseteq A$  or  $E \subseteq B$ .

**Proof.** Assume that  $E \subseteq A \cup B$ ,  $A$  and  $B$  are  $m_X$ -separated sets. Then  $E = E \cap (A \cup B) = (E \cap A) \cup (E \cap B)$ . Since  $A$  and  $B$  are  $m_X$ -separated set,  $A \cap m_X^i Cl(B) = \emptyset$  and  $m_X^i Cl(A) \cap B = \emptyset$ ,  $i = 1, 2$ . Next we will show that  $E \cap A = \emptyset$  or  $E \cap B = \emptyset$ . Suppose that  $E \cap A \neq \emptyset$  and  $E \cap B \neq \emptyset$ . Then

$$\begin{aligned} (E \cap A) \cap m_X^i Cl(E \cap B) &\subseteq (E \cap A) \cap (m_X^i Cl(E) \cap m_X^i Cl(B)) \\ &= E \cap (A \cap m_X^i Cl(B)) \\ &= \emptyset. \end{aligned}$$

Similarly, we have  $m_X^i Cl(E \cap A) \cap (E \cap B) = \emptyset$ . Then  $E \cap A$  and  $E \cap B$  are  $m_X$ -separated sets. Therefore,  $E$  is  $m_X$ -disconnected set. This is contradiction with  $E$  is  $m_X$ -connected set. Thus  $E \cap A = \emptyset$  or  $E \cap B = \emptyset$ . So, we have:

**Case 1.** If  $E \cap A = \emptyset$ , then we have

$$E = (E \cap A) \cup (E \cap B) = E \cap B. \text{ Hence } E \subseteq B.$$

**Case 2.** If  $E \cap B = \emptyset$ , then we have

$$E = (E \cap A) \cup (E \cap B) = E \cap A. \text{ Hence } E \subseteq A.$$

**Theorem 3.14.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $E \subseteq X$  be  $m_X$ -connected set. If  $E \subseteq A \subseteq m_X^i Cl(E)$ , where  $i = 1, 2$ , then  $A$  is  $m_X$ -connected set.

**Proof.** Assume that  $E \subseteq A \subseteq m_X Cl(E)$ . We will show that  $A$  is  $m_X$ -connected set. Suppose that  $A$  is  $m_X$ -disconnected set. Thus, there exist nonempty sets  $C$  and  $D$ , such that  $A = C \cup D$ ,  $C \cap m_X^i Cl(D) = \emptyset$  and  $m_X^i Cl(C) \cap D = \emptyset$ , for  $i = 1, 2$ . Since  $E$  is  $m_X$ -connected set and  $E \subseteq C \cup D$  and Theorem 3.13, we have  $E \subseteq C$  or  $E \subseteq D$ .

**Case 1.** If  $E \subseteq C$ , then we have  $m_X^i Cl(E) \subseteq m_X^i Cl(C)$  and  $m_X^i Cl(E) \cap D = \emptyset$ . Since

$$D \subseteq C \cup D = A \subseteq m_X^i Cl(E), D = m_X^i Cl(E) \cap D = \emptyset.$$

Contradiction with  $D \neq \emptyset$ .

**Case 2.** If  $E \subseteq D$ , then we have  $m_X^i Cl(E) \subseteq m_X^i Cl(D)$  and  $m_X^i Cl(E) \cap C = \emptyset$ . Since

$$C \subseteq C \cup D = A \subseteq m_X^i Cl(E), C = m_X^i Cl(E) \cap C = \emptyset.$$

Contradiction with  $C \neq \emptyset$ .

Therefore,  $A$  is  $m_X$ -connected set.

**Corollary 3.15.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $E \subseteq X$  be  $m_X$ -connected set. If  $E$  is  $m_X$ -connected set, then  $m_X^i Cl(E)$  is  $m_X$ -connected set, where  $i = 1, 2$ .

**Proof.** Since  $E \subseteq m_X^i Cl(E) \subseteq m_X^i Cl(E)$  and Theorem 3.14, we have  $m_X^i Cl(E)$  is  $m_X$ -connected set,  $i = 1, 2$ .

**Theorem 3.16.** Let  $(X, m_X^1, m_X^2)$  be a biminimal structure space and  $E \subseteq X$ . If for any  $x, y \in E$  and  $x \neq y$ , there exists  $m_X$ -connected set  $F \subseteq E$  such that  $x, y \in F$ . Then  $E$  is  $m_X$ -connected set.

**Proof.** Assume that for any  $x, y \in E$  and  $x \neq y$ , there exists  $m_X$ -connected set  $F \subseteq E$  such that  $x, y \in F$ . Next we will show that  $E$  is  $m_X$ -connected set. Suppose that  $E$  is  $m_X$ -disconnected set. Then there exist nonempty sets  $A$  and  $B$  such that  $E = A \cup B$ ,  $A$  and  $B$  are  $m_X$ -separated sets. Since  $A$  and  $B$  are  $m_X$ -separated sets,  $A \neq \emptyset$ ,  $B \neq \emptyset$  and  $A \cap B = \emptyset$ . Then we also have  $x \in A$ ,  $y \in B$  and  $x \neq y$ . Thus,  $x, y \in E$  and by assumption there exists  $m_X$ -connected set  $F \subseteq E$  such that  $x, y \in F$ .

Since  $F \subseteq E = A \cup B$ , by Theorem 3.13 we have  $F \subseteq A$  or  $F \subseteq B$ . Then  $x, y \in A$  or  $x, y \in B$ . This is contradiction with  $A \cap B = \emptyset$ . Therefore,  $E$  is  $m_X$ -connected set.

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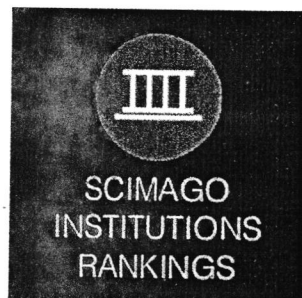
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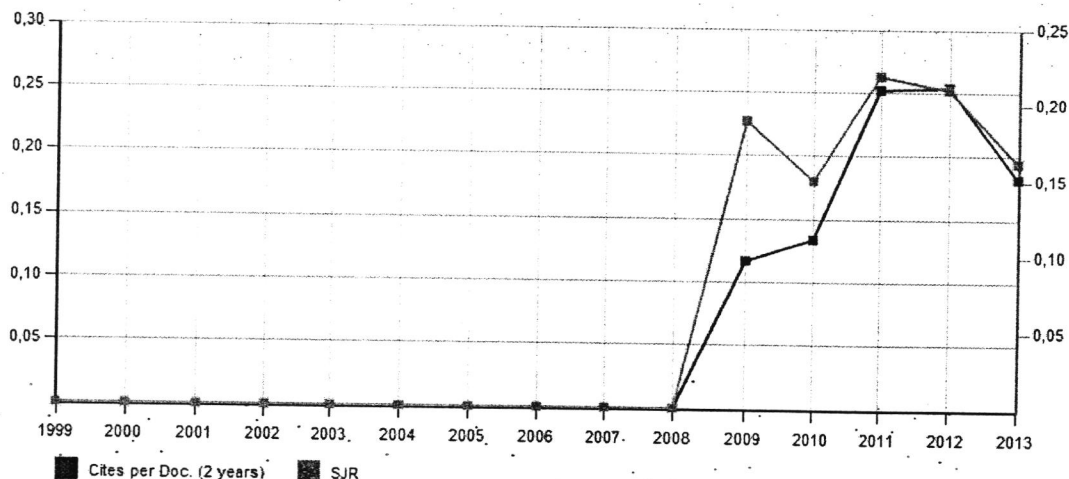
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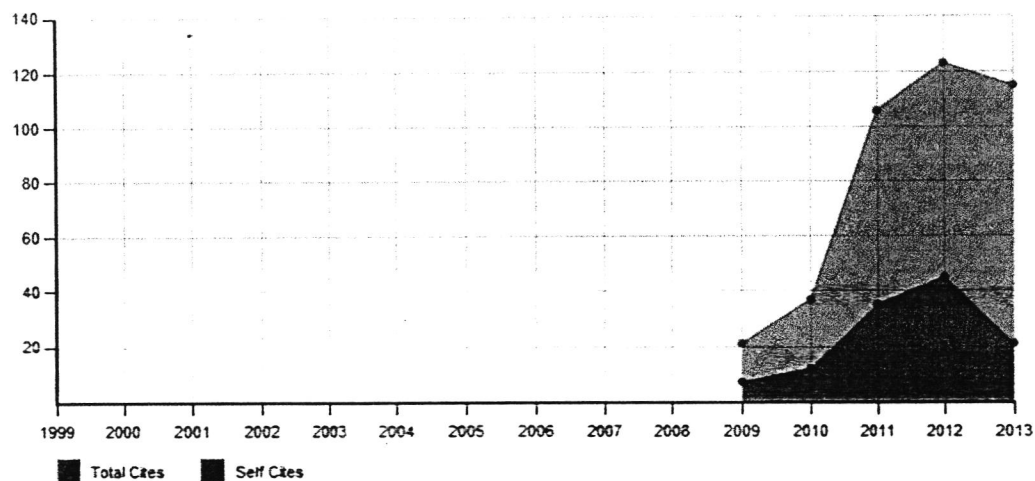


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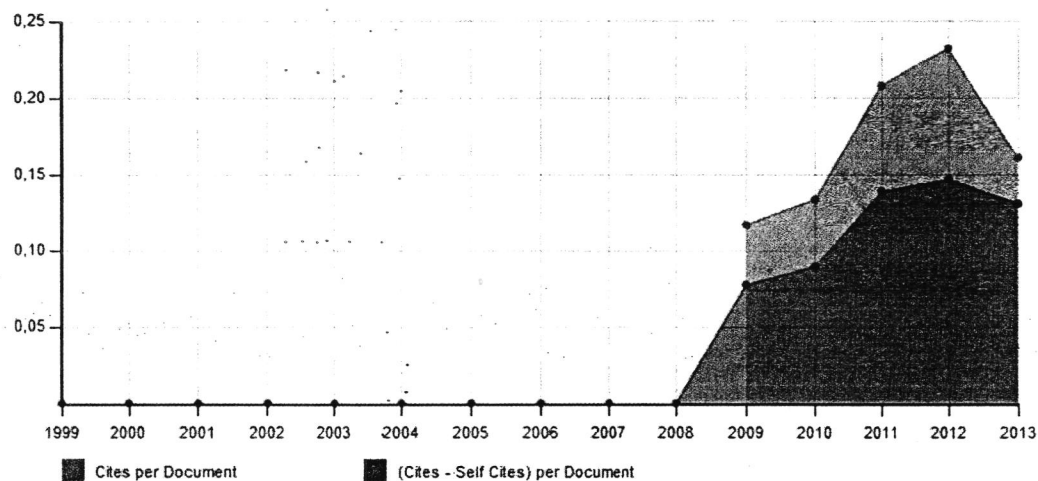
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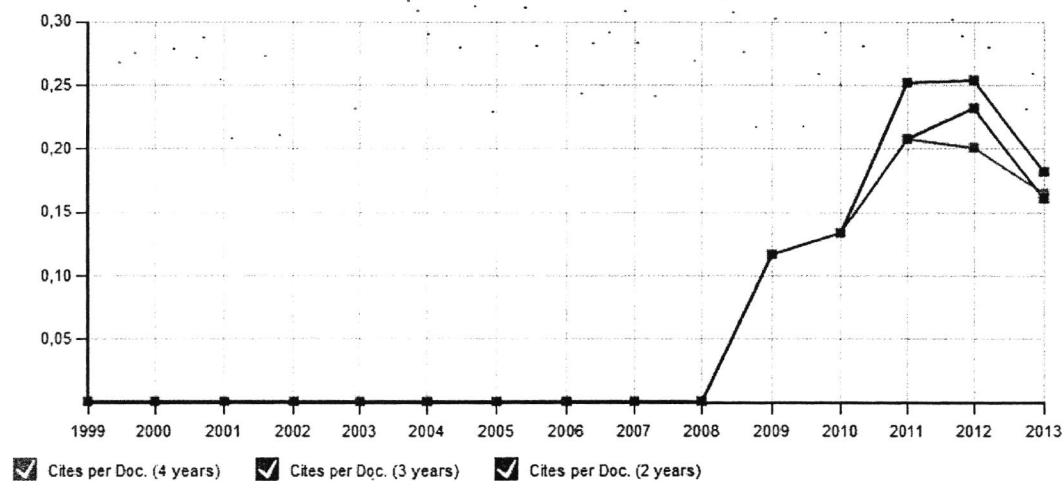
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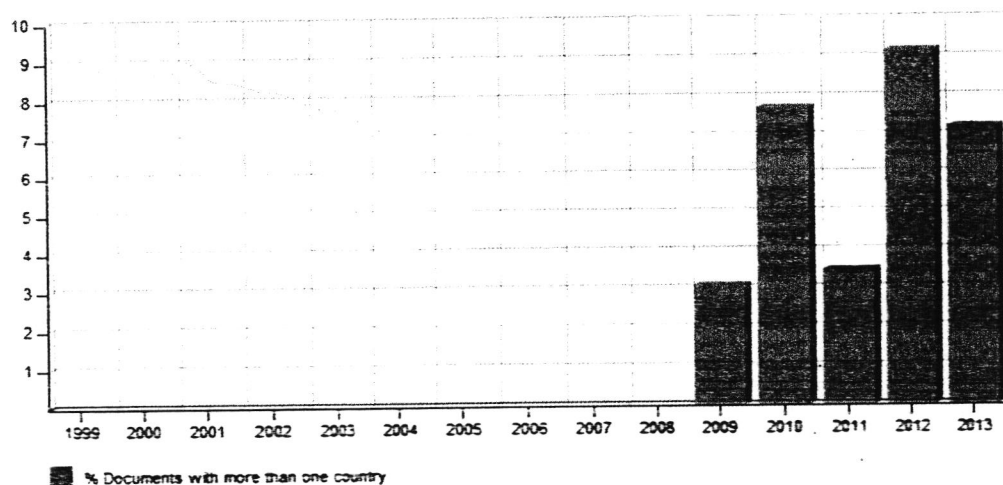
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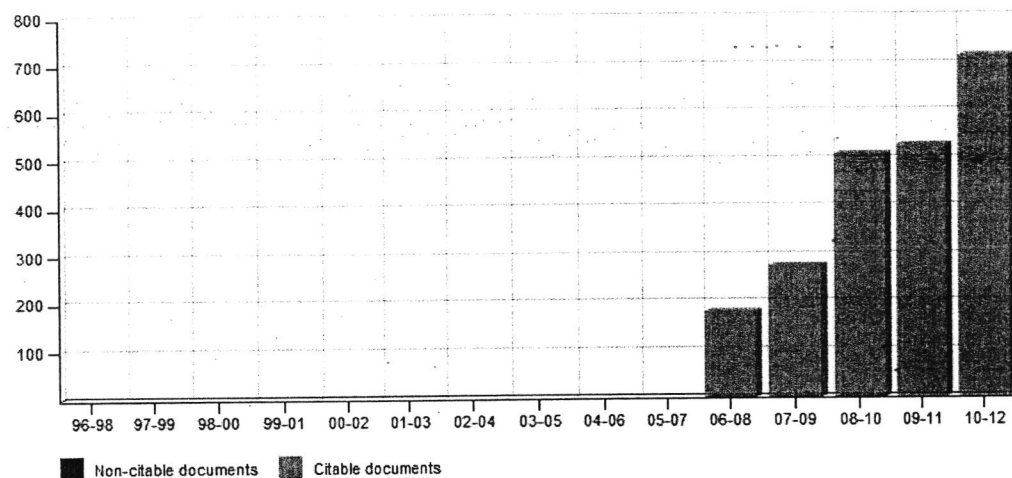
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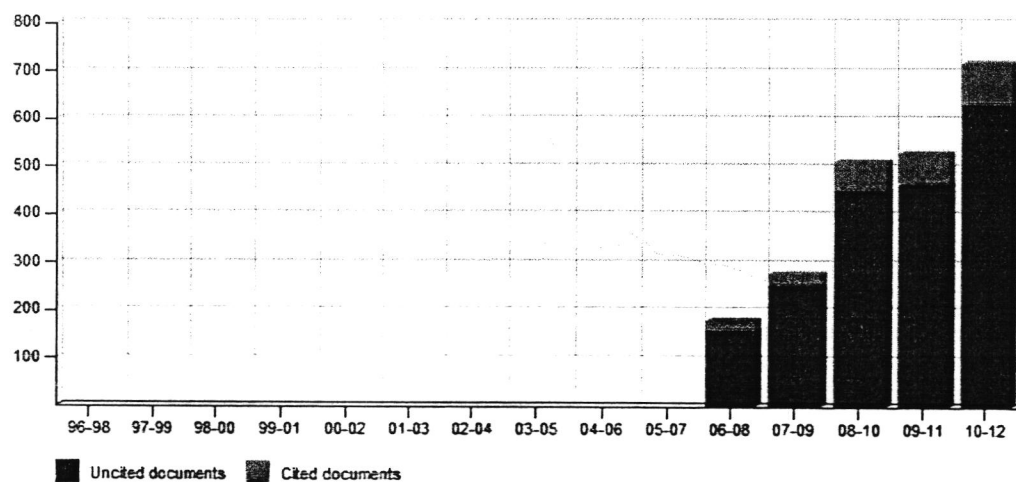
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