

Far East Journal of Mathematical Sciences (FJMS) © 2016 Pushpa Publishing House, Allahabad, India Published Online: May 2016 http://dx.doi.org/10.17654/MS099101513 Volume 99, Number 10, 2016, Pages 1513-1531

ISSN: 0972-0871

(i, j)- m_X - β -BOUNDARY SETS IN BIMINIMAL

STRUCTURE SPACES

Patarawadee Prasertsang and Supunnee Sompong

Department of Sciences

Faculty of Science and Engineering

Kasetsart University

Chalermprakiat Sakon Nakhon Province Campus

Sakon Nakhon 47000

Thailand

e-mail: patarawadee.s@ku.th

Department of Mathematics and Statistics

Faculty of Science and Technology

Sakon Nakhon Rajabhat University

Sakon Nakhon 47000

Thailand

e-mail: s sanpinij@yahoo.com

Abstract

In this paper, we introduce the notion of an (i, j)- m_X -boundary set, in a biminimal structure space and obtain some of its properties besides characterizations. Further, the closure and interior in this setting have also been dealt with.

Received: January 5, 2016; Revised: January 27, 2016; Accepted: February 10, 2016 2010 Mathematics Subject Classification: 54A05.

Keywords and phrases: boundary sets, $(i, j)-m_X-\beta$ -closure, $(i, j)-m_X-\beta$ -interior sets, $(i, j)-m_X-\beta$ -boundary sets, biminimal structure spaces, topology. Communicated by K. K. Azad

1. Introduction

The notion of a bitopological space was introduced by Kelly [4], and that of a minimal structure by Popa and Noiri [10]. Boonpok et al. combined these structures and made preliminary studies in [1-3].

In this paper, we introduce and study the notion of $(i, j)-m_X-\beta$ -closure, $(i, j)-m_X-\beta$ -interior and $(i, j)-m_X-\beta$ -boundary sets in biminimal structure spaces.

2. Preliminaries

Definition 2.1 [8]. A subfamily m_X of the power set P(X) of a nonempty set X is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m_X$ and $X \in m_X$. Each member of m_X is said to be m_X -open and the complement of an m_X -open set is said to be m_X -closed.

Definition 2.2 [5]. Let X be a nonempty set and m_X an *m*-structure on X. For a subset A of X, the m_X -closure of A and m_X -interior of A are defined as follows:

(1) $m_X Cl(A) = \bigcap \{F : A \subseteq F, X \setminus F \in m_X \},$

(2) $m_X Int(A) = \bigcup \{ U : U \subseteq A, U \in m_X \}.$

Lemma 2.3 [5]. Let X be a nonempty set and m_X an m-structure on X. For a subset A and B of X, the following holds:

(1) $m_X Cl(X \setminus A) = X \setminus m_X Int(A)$ and $m_X Int(X \setminus A) = X \setminus m_X Cl(A)$,

(2) If $(X \setminus A) \in m_X$, then $m_X Cl(A) = A$ and if $A \in m_X$, then $m_X Int(A) = A$,

(3) $m_X Cl(\emptyset) = \emptyset$, $m_X Cl(X) = X$, $m_X Int(\emptyset) = \emptyset$ and $m_X Int(X) = X$,

(4) If $A \subseteq B$, then $m_X Cl(A) \subseteq m_X Cl(B)$ and $m_X Int(A) \subseteq m_X Int(B)$,

(5)
$$A \subseteq m_X Cl(A)$$
 and $m_X Int(A) \subseteq A$,

(6) $m_X Cl(m_X Cl(A)) = m_X Cl(A)$ and $m_X Int(m_X Int(A)) = m_X Int(A)$.

Lemma 2.4 [9]. Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then $x \in m_X - Cl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.

Definition 2.5 [3]. Let X be a nonempty set and $m_X^1 m_X^2$ be minimal structures on X. The triple (X, m_X^1, m_X^2) is called a *bispace* (briefly *bi m-space* [7]) or *biminimal structure space* (briefly *bimspace* [1]).

Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subsets of X. The m_X -closure of A and the m_X -interior of A with respect to m_X^i are denoted by $m_X^i Cl(A)$ and $m_X^i Int(A)$, respectively, for i, j = 1, 2 and $i \neq j$.

Definition 2.6 [2]. A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be

(1) (i, j)- m_X -regular-open if $A = m_X^i Int(m_X^j Cl(A))$, for i, j = 1 or 2 and $i \neq j$;

(2) (i, j)- m_X -semi-open if $A \subseteq m_X^i Cl(m_X^j Int(A))$, for i, j = 1 or 2 and $i \neq j$;

(3) (i, j)- m_X -preopen if $A \subseteq m_X^i$ Int $(m_X^j Cl(A))$, for i, j = 1 or 2 and $i \neq j$;

(4) $(i, j)-m_X-\alpha$ -open if $A \subseteq m_X^i Int(m_X^j Cl(m_X^i Int(A)))$, for i, j = 1 or 2 and $i \neq j$;

(5) $(i, j)-m_X-\beta$ -open if $A \subseteq m_X^i Cl(m_X^j Int(m_X^i Cl(A)))$, for i, j = 1 or 2 and $i \neq j$.

Lemma 2.7 [2]. Let (X, m_X^1, m_X^2) be an *m*-space and A be a subset of X. Then

(1) A is (i, j)-m_X-regular-closed if and only if A = mⁱ_XCl(m^j_XInt(A));
(2) A is (i, j)-m_X-semi-closed if and only if mⁱ_XInt(m^j_XCl(A)) ⊆ A;
(3) A is (i, j)-m_X-preclosed if and only if mⁱ_XCl(m^j_XInt(A)) ⊆ A;
(4) A is (i, j)-m_X-α -closed if and only if mⁱ_XCl(m^j_XInt(mⁱ_XCl(A))) ⊆ A;
(5) A is (i, j)-m_X-β-closed if and only if mⁱ_XInt(m^j_XCl(mⁱ_XInt(A))) ⊆ A;

Lemma 2.8 [3]. Let (X, m_X^1, m_X^2) be a biminimal structure space and $\{A_k : k \in \mathcal{K}\}$ a family of subsets of X.

(1) If A_k is $(i, j)-m_X-\alpha$ -open for each $k \in \mathcal{K}$, then $\bigcup_{k \in \mathcal{K}} A_k$ is $(i, j)-m_X-\alpha$ -open.

(2) If A_k is $(i, j)-m_X-\alpha$ -closed for each $k \in \mathcal{K}$, then $\bigcap_{k \in \mathcal{K}} A_k$ is $(i, j)-m_X-\alpha$ -closed.

Definition 2.9 [3]. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X. Then m_X^{ij} - α -closure of A and the m_X^{ij} - α - interior of A are defined as follows:

(1) $m_X^{ij}Cl_{\mathscr{A}}(A) = \bigcap \{F : A \subseteq F, F \text{ is } (i, j) - m_X - \alpha \text{-closed} \};$

(2) m_X^{ij} Int $\mathcal{A}(A) = \bigcup \{U : U \subseteq A, U \text{ is } (i, j) - m_X - \alpha \text{ - open} \}.$

Lemma 2.10 [3]. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X. The following properties hold:

(1) $m_X^{ij}Cl_{\mathscr{A}}(A)$ is $(i, j)-m_X-\alpha$ -closed;

(2) $m_X^{ij} \operatorname{Int}_{\mathscr{A}}(A)$ is $(i, j) - m_X - \alpha - open;$

(3) A is (i, j)- m_X - α - closed if and only if $m_X^{ij}Cl_{\mathscr{A}}(A) = A$;

(4) A is (i, j)- m_X - α - open if and only if m_X^{ij} Int $\mathcal{A}(A) = A$.

Lemma 2.11 [3]. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X. Then, $x \in m_X^{ij}Cl_{\mathscr{A}}(A)$ if and only if $U \cap A \neq \emptyset$ for every $(i, j)-m_X-\alpha$ -openset U containing x.

Lemma 2.12 [3]. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X,

(1)
$$m_X^{ij}Cl_{\mathscr{A}}(X \setminus A) = X \setminus m_X^{ij}Int_{\mathscr{A}}(A);$$

(2) $m_X^{ij}Int_{\mathscr{A}}(X \setminus A) = X \setminus m_X^{ij}Cl_{\mathscr{A}}(A).$

Definition 2.13 [3]. Let (X, m_X^1, m_X^2) be a biminimal structure space and Y be a subset of X. Define minimal structures m_Y^1 and m_Y^2 as follows: $m_Y^1 = \{A \cap Y : A \in m_X^1\}$ and $m_Y^2 = \{B \cap Y : B \in m_X^2\}$. A triple (Y, m_Y^1, m_Y^2) is called a *biminimal structure subspace* (briefly *bim-subspace*) of (X, m_X^1, m_X^2) .

Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) , and let A be a subset of Y. The m_Y -closure and m_Y -interior of A with respect to m_Y^i are denoted by $m_Y^i Cl(A)$ and $m_Y^i Int(A)$, respectively (for i = 1, 2). Then $m_Y^1 Cl(A) = Y \cap m_X^1 Cl(A)$ and $m_Y^2 Cl(A) = Y \cap m_X^2 Cl(A)$.

Proposition 2.14 [3]. Let (Y, m_Y^1, m_Y^2) be a biminimal structure

subspace of (X, m_X^1, m_X^2) and F a subset of Y. If F is $m_X^1 m_X^2$ -closed; then F is $m_Y^1 m_Y^2$ -closed.

3. (i, j)- m_X - β - closure and (i, j)- m_X - β - interior Sets

Definition 3.1. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X. Then m_X^{ij} - β -closure of A and the m_X^{ij} - β -interior of A, where i, j = 1, 2 and $i \neq j$ are defined as follows:

- (1) $m_X^{ij}Cl_{\mathscr{B}}(A) = \bigcap \{F : A \subseteq F, F \text{ is } (i, j) m_X \beta \text{-closed} \};$
- (2) $m_X^{ij} Int_{\mathscr{B}}(A) = \bigcup \{U : U \subseteq A, U \text{ is } (i, j) m_X \beta \text{ open} \}.$

Example 3.2. Let $X = \{1, 2, 3\}$. Define *m*-structures m_X^1 and m_X^2 on the biminimal structure space X as follows: $m_X^1 = \{\emptyset, \{1\}, \{2, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{2\}, \{1, 3\}, X\}$.

Then, by Definition 3.1, we have

$$m_X^{12}Cl_{\mathscr{B}}(\{1,3\}) = \{1,3\}, m_X^{21}Cl_{\mathscr{B}}(\{1,3\}) = X,$$

$$m_X^{12}Int_{\mathscr{B}}(\{1,3\}) = \{1,3\}, \text{ and } m_X^{21}Int_{\mathscr{B}}(\{1,3\}) = \{1,3\}$$

Lemma 3.3. Let (X, m_X^1, m_X^2) be a biminimal structure space and A and B are subsets of X, the following holds:

(1) $m_X^{ij}Cl_{\mathscr{B}}(\varnothing) = \varnothing, \ m_X^{ij}Cl_{\mathscr{B}}(X) = X, \ m_X^{ij}Int_{\mathscr{B}}(\varnothing) = \varnothing, \ m_X^{ij}Int_{\mathscr{B}}(X) = X,$

(2) $A \subseteq m_X^{ij} Cl_{\mathscr{B}}(A)$ and $m_X^{ij} Int_{\mathscr{B}}(A) \subseteq A$,

(3) If $A \subseteq B$, then $m_X^{ij}Cl_{\mathscr{B}}(A) \subseteq m_X^{ij}Cl_{\mathscr{B}}(B)$ and $m_X^{ij}Int_{\mathscr{B}}(A) \subseteq m_X^{ij}Int_{\mathscr{B}}(B)$.

Proof. Assume that (X, m_X^1, m_X^2) is a biminimal structure space, A and B are subsets of X.

(1) Since \emptyset and X are both (i, j)- m_X - β -closed and (i, j)- m_X - β -open. So $m_X^{ij}Cl_{\mathscr{B}}(\emptyset) = \emptyset$, $m_X^{ij}Cl_{\mathscr{B}}(X) = X$, $m_X^{ij}Int_{\mathscr{B}}(\emptyset) = \emptyset$, $m_X^{ij}Int_{\mathscr{B}}(X) = X$,

(2) and (3). It follows immediately from Definition 3.1.

Lemma 3.4. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X. The following properties hold:

(1) $m_X^{ij}Cl_{\mathscr{B}}(A)$ is $(i, j)-m_X-\beta$ -closed;

(2) $m_X^{ij} Int_{\mathscr{B}}(A)$ is $(i, j)-m_X-\beta$ -open;

(3) A is (i, j)- m_X - β -closed if and only if $m_X^{ij}Cl_{\mathscr{B}}(A) = A$;

(4) A is (i, j)- m_X - β -open if and only if m_X^{ij} Int $\mathscr{B}(A) = A$.

Proof. (1) and (2). They are obvious by Definition 3.1.

(3) This follows from Definition 3.1 and (1) immediately.

(4) This follows from Definition 3.1 and (2) immediately.

Lemma 3.5. Let (X, m_X^1, m_X^2) be a biminimal structure space and A and B are subsets of X, the following holds:

(1) If A and B are (i, j)- m_X - β -closed, then $A \cap B$ is (i, j)- m_X - β -closed.

(2) If A and B are (i, j)- m_X - β -open, then $A \cup B$ is (i, j)- m_X - β -open.

Proof. Assume that (X, m_X^1, m_X^2) is a biminimal structure space, A and B are subsets of X.

(1) Suppose that A and B are (i, j)- m_X - β -closed; then $m_X^{ij}Cl_{\mathscr{B}}(A) = A$

and $m_X^{ij}Cl_{\mathscr{B}}(B) = B$, respectively. Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, we obtain $m_X^{ij}Cl_{\mathscr{B}}(A \cap B) \subseteq m_X^{ij}Cl_{\mathscr{B}}(A)$ and $m_X^{ij}Cl_{\mathscr{B}}(A \cap B) \subseteq m_X^{ij}Cl_{\mathscr{B}}(B)$. It means that $m_X^{ij}Cl_{\mathscr{B}}(A \cap B) \subseteq A \cap B$. On the other hand, $A \cap B \subseteq m_X^{ij}Cl_{\mathscr{B}}(A \cap B)$. Therefore, $m_X^{ij}Cl_{\mathscr{B}}(A \cap B) = A \cap B$. Consequently, $A \cap B$ is (i, j)- m_X - β -closed.

(2) Similar to 1.

1520

Remark 3.6. (1) The union of two $(i, j)-m_X-\beta$ -closed is not a $(i, j)-m_X-\beta$ -closed in general. For instance, by Example 3.2, {2} and {3} are $(1, 2)-m_X-\beta$ -closed, whereas {2, 3} is not $(1, 2)-m_X-\beta$ -closed. Moreover, {1} and {3} are $(1, 2)-m_X-\beta$ -closed, whereas {1, 3} is not $(1, 2)-m_X-\beta$ -closed.

(2) The intersection of two $(i, j)-m_X-\beta$ -open is not a $(i, j)-m_X-\beta$ -open in general. For instance, by Example 3.2, $\{1, 2\}$ and $\{1, 3\}$ are $(1, 2)-m_X-\beta$ closed, whereas $\{1\}$ is not $(1, 2)-m_X-\beta$ -closed: Moreover, $\{1, 2\}$ and $\{2, 3\}$ are $(2, 1)-m_X-\beta$ -closed, whereas $\{2\}$ is not $(2, 1)-m_X-\beta$ -closed.

Lemma 3.7. Let (X, m_X^1, m_X^2) be a biminimal structure space and $\{B_k : k \in \mathcal{K}\}$ a family of subsets of X.

(1) If B_k is $(i, j)-m_X-\beta$ -open for each $k \in \mathcal{K}$, then $\bigcup_{k \in \mathcal{K}} B_k$ is $(i, j)-m_X-\beta$ -open.

(2) If B_k is $(i, j)-m_X-\beta$ -closed for each $k \in \mathcal{K}$, then $\bigcap_{k \in \mathcal{K}} B_k$ is $(i, j)-m_X-\beta$ -closed.

Proof. (1) Assume that B_k is $(i, j) \cdot m_X \cdot \beta \cdot open$ for each $k \in \mathcal{K}$. It means that $B_k \subseteq m_X^i Cl(m_X^j Int(m_X^i Cl(B_k)))$, for each $k \in \mathcal{K}$ which i, j = 1 or 2 and $i \neq j$. Since $B_p \subseteq \bigcup_{k \in \mathcal{K}} B_k$, for all $p \in \mathcal{K}$. We have that

 $(i, j) - m_X - \beta$ -boundary Sets in Biminimal Structure Spaces 1521 $m_X^i Cl(m_X^j Int(m_X^i Cl(B_p))) \subseteq m_X^i Cl\left(m_X^j Int\left(m_X^i Cl\left(\bigcup_{k \in \mathscr{K}} B_k\right)\right)\right)$, for all $p \in \mathscr{K}$. Let us consider

$$\bigcup_{k \in \mathscr{K}} B_k \subseteq \bigcup_{p \in \mathscr{K}} m_X^i Cl(m_X^j Int(m_X^i Cl(B_p)))$$
$$\subseteq m_X^i Cl\left(m_X^j Int\left(m_X^i Cl\left(\bigcup_{k \in \mathscr{K}} B_k\right)\right)\right) \text{ for all } p \in \mathscr{K}$$

That is, $\bigcup_{k \in \mathscr{K}} B_k \subseteq m_X^i Cl\left(m_X^j Int\left(m_X^i Cl\left(\bigcup_{k \in \mathscr{K}} B_k\right)\right)\right)$.

Therefore, $\bigcup_{k \in \mathscr{K}} B_k$ is $(i, j) - m_X - \beta$ -open for each $k \in \mathscr{K}$.

(2) This follows from (1) immediately.

Lemma 3.8. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X. Then $x \in m_X^{ij}Cl_{\mathscr{B}}(A)$ if and only if $V \cap A \neq \emptyset$ for every (i, j)- m_X - β -open set V containing x.

Proof. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X.

(⇒) Assume that $x \in m_X^{ij}Cl_{\mathscr{B}}(A)$. Thus, $x \in F$, for each F, which is $(i, j)-m_X-\beta$ -closed and $A \subseteq F$. Suppose that there exists $(i, j)-m_X-\beta$ -open set, $x \in V$ and $V \cap A = \emptyset$. That is, $V \subseteq X \setminus A$, or $A \subseteq X \setminus V$. Since $x \notin X \setminus V$, which is $(i, j)-m_X-\beta$ -closed, $x \notin m_X^{ij}Cl_{\mathscr{B}}(A)$, which disagree with the assumption. It follows that $V \cap A \neq \emptyset$ for every $(i, j)-m_X-\beta$ -open set V containing X.

(\Leftarrow) Suppose that $V \cap A \neq \emptyset$ for every $(i, j) \cdot m_X \cdot \beta$ -open set V containing x. Assume that $x \notin m_X^{ij} Cl_{\mathscr{B}}(A)$. It follows that there exists F, which is $(i, j) \cdot m_X \cdot \beta$ -closed and $A \subseteq F$ such that $x \notin F$. Then, $x \in X \setminus F$.

Since $(X \setminus F) \cap F = \emptyset$ and $A \subseteq F$, thus $(X \setminus F) \cap F = \emptyset$, but $X \setminus F$ is $(i, j) \cdot m_X \cdot \beta$ -open set such that $(X \setminus F) \cap F \neq \emptyset$, then $x \in m_X^{ij} Cl_{\mathscr{B}}(A)$. \Box

Lemma 3.9. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X,

(1) $m_X^{ij} Int_{\mathscr{B}}(X \setminus A) = X \setminus m_X^{ij} Cl_{\mathscr{B}}(A);$

1522

(2) $m_X^{ij}Cl_{\mathscr{B}}(X \setminus A) = X \setminus m_X^{ij}Int_{\mathscr{B}}(A).$

Proof. Assume (X, m_X^1, m_X^2) is a biminimal structure space and A a subset of X.

(1) Suppose that $x \notin X \setminus m_X^{ij} Cl_{\mathscr{B}}(A)$. Thus, $x \in Xm_X^{ij} Cl_{\mathscr{B}}(A)$. By Lemma 3.8, for every $(i, j) - m_X - \beta$ open V such that $x \in V$ and $V \cap A \neq \emptyset$. That is, $x \notin m_X^{ij} Int_{\mathscr{B}}(X \setminus A)$. It follows that $m_X^{ij} Int_{\mathscr{B}}(X \setminus A) \subseteq X \setminus m_X^{ij} Cl_{\mathscr{B}}(A)$.

Conversely, since $m_X^{ij}Cl_{\mathscr{B}}(A)$ is $(i, j)-m_X-\beta$ -closed, $X \setminus m_X^{ij}Cl_{\mathscr{B}}(A)$ is $(i, j)-m_X-\beta$ -open. Since $A \subseteq m_X^{ij}Cl_{\mathscr{B}}(A)$, we have $X \setminus m_X^{ij}Cl_{\mathscr{B}}(A) \subseteq X \setminus A$. Furthermore, by Definition 3.1, we obtain $X \setminus m_X^{ij}Cl_{\mathscr{B}}(A) \subseteq m_X^{ij}Int_{\mathscr{B}}(X \setminus A)$. Consequently, $m_X^{ij}Int_{\mathscr{B}}(X \setminus A) = X \setminus m_X^{ij}Cl_{\mathscr{B}}(A)$.

(2) This follows from (1) immediately.

Lemma 3.10. Let (X, m_X^1, m_X^2) be a biminimal structure space and A and B are subsets of X.

(1) If A and B are (i, j)- m_X - β -closed, then

 $m_X^{ij}Cl_{\mathscr{B}}(A \cap B) = m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(B).$

(2) If A and B are (i, j)- m_X - β -open, then

 $m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A \cup B) = m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A) \cup m_X^{ij} \operatorname{Int}_{\mathscr{B}}(B).$

(3) If A and B are $(i, j)-m_X-\beta$ -closed and $A \cup B$ is $(i, j)-m_X-\beta$ closed, then $m_X^{ij}Cl_{\mathscr{B}}(A \cup B) = m_X^{ij}Cl_{\mathscr{B}}(A) \cup m_X^{ij}Cl_{\mathscr{B}}(B)$.

(4) If A and B are (i, j)- m_X - β -open and $A \cap B$ is (i, j)- m_X - β -open, then m_X^{ij} Int_B $(A \cap B) = m_X^{ij}$ Int_B $(A) \cap m_X^{ij}$ Int_B(B).

Proof. Assume that (X, m_X^1, m_X^2) is a biminimal structure space and A and B are subsets of X.

(1) Let A and B be $(i, j)-m_X-\beta$ -closed. Then, $m_X^{ij}Cl_{\mathscr{B}}(A) = A$ and $m_X^{ij}Cl_{\mathscr{B}}(B) = B$. By Lemma 3.5 (1), $A \cap B$ is $(i, j)-m_X-\beta$ -closed. It follows that $m_X^{ij}Cl_{\mathscr{B}}(A \cap B) = A \cap B = m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(B)$.

(2) Similar to (1), $m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A \cup B) = A \cup B = m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A) \cup m_X^{ij} \operatorname{Int}_{\mathscr{B}}(B).$

(3) Let A, B be (i, j)- m_X - β -closed and $A \cup B$ a (i, j)- m_X - β -closed. We have that $m_X^{ij}Cl_{\mathscr{B}}(A) = A$, $m_X^{ij}Cl_{\mathscr{B}}(B) = B$ and $m_X^{ij}Cl_{\mathscr{B}}(A \cup B) = A \cup B$. Therefore, $m_X^{ij}Cl_{\mathscr{B}}(A \cup B) = m_X^{ij}Cl_{\mathscr{B}}(A) \cup m_X^{ij}Cl_{\mathscr{B}}(B)$.

(4) Similar to (3), $m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A \cap B) = m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A) \cap m_X^{ij} \operatorname{Int}_{\mathscr{B}}(B).$

4. (i, j)- m_X - β -boundary Sets

Based on the general topology [6], the boundary of a subset A of a topological space X are the set of points which can be approached both from the closure of A and from the closure of the outside of A. Therefore, in this section, we define the new definitions and construct their properties of (i, j)-m_X- β -boundary sets in biminimal structure spaces.

Definition 4.1. Let (X, m_X^1, m_X^2) be a biminimal structure space, A be a subset of X and $x \in X$. Then, x is called (i, j)- m_X - β -boundary point of A if

 $x \in m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus A)$. We denote that the set of all $(i, j)-m_X-\beta$ -boundary point of A by $m_X^{ij}Bdr_{\mathscr{B}}(A)$, where i, j = 1, 2 and $i \neq j$.

From the Definition 4.1, $m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus A).$

Example 4.2. Let $X = \{1, 2, 3\}$. Define *m*-structures m_X^1 and m_X^2 on the biminimal structure space X as follows: $m_X^1 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{3\}, \{1, 2\}, \{2, 3\}, X\}$. By the Definition 4.1, we obtain that $m_X^{ij}Bdr_{\mathscr{B}}(\{1, 3\}) = m_X^{ij}Cl_{\mathscr{A}}(\{1, 3\}) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus \{1, 3\})$.

Consequently, $m_X^{12}Bdr_{\mathscr{B}}(\{1, 3\}) = \{1, 2\}$ and $m_X^{21}Bdr_{\mathscr{B}}(\{1, 3\}) = \emptyset$.

Lemma 4.3. Let (X, m_X^1, m_X^2) be a biminimal structure space and A be a subset of X. Then $m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Bdr_{\mathscr{B}}(X \setminus A)$, where i, j = 1, 2 and $i \neq j$.

Proof. For i, j = 1, 2 and $i \neq j$. By Definition 4.1, $m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus A)$ and also $m_X^{ij}Bdr_{\mathscr{B}}(X \setminus A) = m_X^{ij}Cl_{\mathscr{B}}(X \setminus A) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus (X \setminus A)) = m_X^{ij}Cl_{\mathscr{B}}(X \setminus A) \cap m_X^{ij}Cl_{\mathscr{B}}(A)$. Consequently,

 $m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Bdr_{\mathscr{A}}(X \setminus A)$, where i, j = 1, 2 and $i \neq j$.

Lemma 4.4. Let (X, m_X^1, m_X^2) be a biminimal structure space, A be a subset of X. Then for any i, j = 1, 2 and $i \neq j$, the following statements are hold:

- (1) $m_X^{ij} Bdr_{\mathscr{B}}(A)$ is $(i, j)-m_X-\beta$ -closed.
- (2) $m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Cl_{\mathscr{B}}(A) \setminus m_X^{ij}Int_{\mathscr{B}}(A).$
- (3) $m_Y^{ij} Bdr_{\mathscr{B}}(A) \cap m_Y^{ij} Int_{\mathscr{B}}(A) = \emptyset$.

(4)
$$m_X^{ij} Bdr_{\mathscr{B}}(A) \cap m_X^{ij} Int_{\mathscr{B}}(X \setminus A) = \emptyset$$
.

(5)
$$m_X^{ij}Cl_{\mathscr{B}}(A) = m_X^{ij}Bdr_{\mathscr{B}}(A) \cup m_X^{ij}Int_{\mathscr{B}}(A).$$

(6) $X = m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A) \cup m_X^{ij} \operatorname{Bdr}_{\mathscr{B}}(A) \cup m_X^{ij} \operatorname{Int}_{\mathscr{B}}(X \setminus A)$ is a pairwise disjoint union.

(7)
$$m_X^{ij} Bdr_{\mathscr{B}}(m_X^{ij} Int_{\mathscr{B}}(A)) \subseteq m_X^{ij} Bdr_{\mathscr{B}}(A).$$

(8)
$$m_X^{ij}Bdr_{\mathscr{B}}(m_X^{ij}Cl_{\mathscr{B}}(A)) \subseteq m_X^{ij}Bdr_{\mathscr{B}}(A).$$

Proof. Assume that (X, m_X^1, m_X^2) is a biminimal structure space and A, B are subsets of X.

(1) Since $m_X^{ij}Cl_{\mathscr{B}}(A)$ and $m_X^{ij}Cl_{\mathscr{B}}(X \setminus A)$ are $(i, j)-m_X-\beta$ -closed and by Lemma 3.5, we obtain that $m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus A)$ is $(i, j)-m_X-\beta$ closed. Finally, by Definition 3.1, we then have $m_X^{ij}Bdr_{\mathscr{B}}(A)$ is $(i, j)-m_X-\beta$ - β -closed.

$$(2) \ m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Cl_{\mathscr{B}}(A) \cap m_X^{ij}Cl_{\mathscr{B}}(X \setminus A)$$
$$= m_X^{ij}Cl_{\mathscr{B}}(A) \cap (X \setminus m_X^{ij}Int_{\mathscr{B}}(A))$$
$$= m_Y^{ij}Cl_{\mathscr{B}}(A) \setminus m_Y^{ij}Int_{\mathscr{B}}(A).$$

(3) By (2), we have that $m_X^{ij}Bdr_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(A) = \emptyset$.

(4) By Lemma 4.3 and (2), we have $m_X^{ij}Bdr_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(X \setminus A) = \emptyset$.

(5) It is clearly by (2).

(6) $m_X^{ij} Int_{\mathscr{B}}(A) \cup m_X^{ij} Bdr_{\mathscr{B}}(A) \cup m_X^{ij} Int_{\mathscr{B}}(X \setminus A)$

= X.

$$= m_X^{ij} Cl_{\mathscr{B}}(A) \cup m_X^{ij} Int_{\mathscr{B}}(X \setminus A)$$

By (3) and (4), we obtain that $m_X^{ij}Bdr_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(A) = \emptyset$ and $m_X^{ij}Bdr_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(X \setminus A) = \emptyset$. In order to complete the proof, we need to show that $m_X^{ij}Int_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(X \setminus A) = \emptyset$. As a result of $m_X^{ij}Int_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(A) = \emptyset$. As a result of $m_X^{ij}Int_{\mathscr{B}}(A) \cap m_X^{ij}Int_{\mathscr{B}}(A) = \emptyset$. Therefore,

$$X = m_X^{ij} \operatorname{Int}_{\mathscr{B}}(A) \cup m_X^{ij} Bdr_{\mathscr{B}}(A) \cup m_X^{ij} \operatorname{Int}_{\mathscr{B}}(X \setminus A)$$

is a pairwise disjoint union. The proof is completed.

(7) By (2), $m_X^{ij}Bdr_{\mathscr{B}}(m_X^{ij}Int_{\mathscr{B}}(A)) = m_X^{ij}Cl_{\mathscr{B}}(m_X^{ij}Int_{\mathscr{B}}(A)) \setminus m_X^{ij}Int_{\mathscr{B}}$ $(m_X^{ij}Int_{\mathscr{B}}(A))$. Since, $m_X^{ij}Cl_{\mathscr{B}}(m_X^{ij}Int_{\mathscr{B}}(A)) \subseteq m_X^{ij}Cl_{\mathscr{B}}(A)$ and $m_X^{ij}Int_{\mathscr{B}}(A)$ $(m_X^{ij}Int_{\mathscr{B}}(A)) = m_X^{ij}Int_{\mathscr{B}}(A)$. Thus, $m_X^{ij}Bdr_{\mathscr{B}}(m_X^{ij}Int_{\mathscr{B}}(A)) \subseteq m_X^{ij}Cl_{\mathscr{B}}(A) \setminus m_X^{ij}Int_{\mathscr{B}}(A) = m_X^{ij}Bdr_{\mathscr{B}}(A)$.

(8) By (2), $m_X^{ij}Bdr_{\mathscr{B}}(m_X^{ij}Cl_{\mathscr{B}}(A)) = m_X^{ij}Cl_{\mathscr{B}}(m_X^{ij}Cl_{\mathscr{B}}(A)) \setminus m_X^{ij}Int_{\mathscr{B}}$ $(m_X^{ij}Cl_{\mathscr{B}}(A)).$ Since, $m_X^{ij}Cl_{\mathscr{B}}(m_X^{ij}Cl_{\mathscr{B}}(A)) = m_X^{ij}Cl_{\mathscr{B}}(A)$ and $m_X^{ij}Int_{\mathscr{B}}(A)$ $\subseteq m_X^{ij}Int_{\mathscr{B}}(m_X^{ij}Cl_{\mathscr{B}}(A)).$ So,

$$m_X^{ij}Bdr_{\mathscr{B}}(m_X^{ij}Cl_{\mathscr{B}}(A)) \subseteq m_X^{ij}Cl_{\mathscr{B}}(A) \setminus m_X^{ij}Int_{\mathscr{B}}(A) = m_X^{ij}Bdr_{\mathscr{B}}(A). \quad \Box$$

By Example 4.2, we have $m_X^{12}Bdr_{\mathscr{B}}(\{2\}) = \{1, 2\}$, and $m_X^{21}Bdr_{\mathscr{B}}(\{1, 2\}) = \{3\}$, which are $(1, 2)-m_X-\beta$ -closed and $(2, 1)-m_X-\beta$ -closed, respectively. Furthermore,

 $m_X^{12} Bdr_{\mathscr{B}}(m_X^{12} Int_{\mathscr{B}}(\{2\})) \subseteq m_X^{12} Bdr_{\mathscr{B}}(\{2\}),$ $m_X^{21} Bdr_{\mathscr{B}}(m_X^{21} Int_{\mathscr{B}}(\{3\})) \subseteq m_X^{21} Bdr_{\mathscr{B}}(\{3\}),$ $m_X^{12} Bdr_{\mathscr{B}}(m_X^{12} Cl_{\mathscr{B}}(\{1, 2\})) \subseteq m_X^{12} Bdr_{\mathscr{B}}(\{1, 2\}),$

and

$$m_X^{21}Bdr_{\mathscr{B}}(m_X^{21}Cl_{\mathscr{B}}(\{2,3\})) \subseteq m_X^{21}Bdr_{\mathscr{B}}(\{2,3\}).$$

Theorem 4.5. Let (X, m_X^1, m_X^2) be a biminimal structure space, A be a subset of X. Then, for any i, j = 1, 2 and $i \neq j$, we obtain that

(1) A is $(i, j)-m_X-\beta$ -closed if and only if it contains all of its $(i, j)-m_X-\beta$ -boundary point.

(2) A is $(i, j)-m_X-\beta$ -open if and only if the complement of A contains all of $(i, j)-m_X-\beta$ -boundary point of A.

Proof. Let (X, m_X^1, m_X^2) be a biminimal structure space, A be a subset of X.

(1) (\Rightarrow) Assume that A is (i, j)- m_X - β -closed. That is, $m_X^{ij}Cl_{\mathscr{B}}(A) = A$. Next, we want to show that $m_X^{ij}Bdr_{\mathscr{B}}(A) \cap (X \setminus A) = \emptyset$. By Definition 4.1, we have

$$m_X^{ij} \underline{B} dr_{\mathscr{B}}(A) \cap (X \setminus A) = (m_X^{ij} Cl_{\mathscr{B}}(A) \cap m_X^{ij} Cl_{\mathscr{B}}(X \setminus A)) \cap (X \setminus A)$$
$$= A \cap (X \setminus A)$$
$$= \emptyset$$

Therefore, $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq A$.

 $(\Leftarrow) \text{ Let } m_X^{ij} Bdr_{\mathscr{B}}(A) \text{ be a subset of } A. \text{ Then, } m_X^{ij} Bdr_{\mathscr{B}}(A) \cap (X \setminus A)$ $= \emptyset. \text{ Since } X \setminus A \subseteq m_X^{ij} Cl_{\mathscr{B}}(X \setminus A), \ m_X^{ij} Cl_{\mathscr{B}}(A) \cap (X \setminus A) = \emptyset, \text{ and finally,}$ $m_X^{ij} Cl_{\mathscr{B}}(A) \subseteq A. \text{ On the other hand, } A \subseteq m_X^{ij} Cl_{\mathscr{B}}(A). \text{ It follows that}$ $m_X^{ij} Cl_{\mathscr{B}}(A) = A. \text{ Moreover, } A \text{ is } (i, j) - m_X - \beta \text{ -closed.}$

(2) This follows by Lemma 4.3 and (1).

Example 4.6. Let $X = \{1, 2, 3\}$. Define *m*-structures m_X^1 and m_X^2 on the biminimal structure space X as follows: $m_X^1 = \{\emptyset, \{2\}, \{1, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{2, 3\}, X\}$.

It is clear that $m_X^{12}Bdr_{\mathscr{B}}(\{2,3\}) = \emptyset \subseteq \{2,3\}$. Whereas, $m_X^{12}Bdr_{\mathscr{B}}(\{1,3\}) = \{2\} \not\subseteq \{1,3\}$. Likewise, $m_X^{21}Bdr_{\mathscr{B}}(\{1,3\}) = \emptyset \subseteq \{1,3\}$. Whereas, $m_X^{21}Bdr_{\mathscr{B}}(\{1\}) = \{1,3\} \not\subseteq \{1\}$. Since, $\{2,3\}$ and $(\{1,3\})$ are (1,2)- m_X - β -closed and (2,1)- m_X - β -closed, respectively. On the other hand, $m_X^{12}Bdr_{\mathscr{B}}(\{2\}) = \{2\} \not\subseteq \{1,3\}$, and $m_X^{21}Bdr_{\mathscr{B}}(\{1\}) = \{1,3\} \not\subseteq \{2,3\}$, because of $\{2\}$ and $\{1\}$ are not (1, 2)- m_X - β -open and (2, 1)- m_X - β -open.

In order to show that $m_X^{ij}Bdr_{\mathscr{B}}(A)$ is equal to \emptyset where i, j = 1, 2 and $i \neq j$. We need some conditions to derive them such as in Example 4.6, we can see that $m_X^{12}Bdr_{\mathscr{B}}(\{1, 2\}) = \{2\}$, on while $m_X^{21}Bdr_{\mathscr{B}}(\{2\}) = \emptyset$. All conditions to approach our purpose are found in the next theorem.

Theorem 4.7. Let (X, m_X^1, m_X^2) be a biminimal structure space, A be a subset of X. Then, $m_X^{ij}Bdr_{\mathscr{B}}(A) = \emptyset$ if and only if A is $(i, j)-m_X-\beta$ -closed and $(i, j)-m_X-\beta$ -open where i, j = 1, 2 and $i \neq j$.

Proof. Let (X, m_X^1, m_X^2) be a biminimal structure space and A be a subset of X.

(⇒) Assume that $m_X^{ij}Bdr_{\mathscr{B}}(A) = \emptyset$. Thus, $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq A$ and $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq X \setminus A$. By Theorem 4.5, A is $(i, j)-m_X-\beta$ -closed and $(i, j)-m_X-\beta$ -open.

(\Leftarrow) Assume that A is (i, j)- m_X - β -closed and (i, j)- m_X - β -open. By Theorem 4.5, we also have $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq A$ and $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq X \setminus A$. As a result, $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq A \cap (X \setminus A)$, and also $m_X^{ij}Bdr_{\mathscr{B}}(A) = \emptyset$. \Box

From Example 4.6, since $\{1\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$ are both (1, 2)- m_X - β closed and (1, 2)- m_X - β -open. Thus, the boundary of those sets are equal to \emptyset . Similarly, $m_X^{21}Bdr_{\mathscr{B}}(\{2\}) = m_X^{21}Bdr_{\mathscr{B}}(\{3\}) = \emptyset$, since $\{2\}$ and $\{3\}$ are both (2, 1)- m_X - β -closed and (2, 1)- m_X - β -open.

Definition 4.8. Let (X, m_X^1, m_X^2) be a biminimal structure spaces and Y be a subset of X. Define m_Y^1 and m_Y^2 as follows: $m_Y^1 = \{A \cap Y : A \in m_X^1\}$ and $m_Y^2 = \{B \cap Y : B \in m_X^2\}$. A triple (Y, m_Y^1, m_Y^2) is called a *biminimal structure subspace* of (X, m_X^1, m_X^2) .

Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) , and let A be a subset of Y. The (i, j)- m_Y - β -closure and (i, j)- m_Y - β -interior of A with respect to m_Y^{ij} are denoted by $m_Y^{ij}Cl_{\mathscr{B}}(A)$ and $m_Y^{ij}Int_{\mathscr{B}}(A)$, respectively (for i = 1, 2 and $i \neq j$). Then $m_Y^{ij}Cl_{\mathscr{B}}(A) = Y \cap m_X^{ij}Cl_{\mathscr{B}}(A)$ and $m_Y^{ij}Int_{\mathscr{B}}(A) = Y \cap m_X^{ij}Int_{\mathscr{B}}(A)$. Moreover, we denote that $m_Y^{ij}Bdr_{\mathscr{B}}(A)$ $= Y \cap m_X^{ij}Bdr_{\mathscr{B}}(A)$.

Lemma 4.9. Let (Y, m_X^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) and A a subset of Y. If A is $(i, j)-m_X-\beta$ -closed, then A is $(i, j)-m_Y-\beta$ -closed.

Proof. Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace a (X, m_X^1, m_X^2) and A a subset of Y. Suppose that A is $(i, j)-m_X-\beta$ -closed, then $m_X^{ij}Cl_{\mathscr{B}}(A) = A$. Consider, $m_Y^{ij}Cl_{\mathscr{B}}(A) = m_X^{ij}Cl_{\mathscr{B}}(A) \cap Y = A \cap Y = A$. Therefore, A is $(i, j)-m_Y-\beta$ -closed.

From Example 4.6, let $Y = \{1, 3\}$, we have that $m_Y^1 = \{\emptyset, \{1, 3\}\}$ and $m_Y^2 = \{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$. It follows that $\emptyset, \{1\}, \{3\}, \{1, 3\}$ is $(1, 2)-m_Y-\beta$ -

closed; and \emptyset , {1, 3} is (2, 1)- m_Y - β -closed; since, \emptyset , {1}, {3}, {1, 3} is (1, 2)- m_X - β -closed; and \emptyset , {1, 3} is (2, 1)- m_X - β -closed, respectively.

Remark 4.10. (1) The converse of Lemma 4.9 is not true.

(2) If A is not a subset of Y and $(i, j)-m_X-\beta$ -closed, then A is not necessary be $(i, j)-m_Y-\beta$ -closed.

From Example 4.6, $\{1, 3\}$ is $(1, 2)-m_Y-\beta$ -closed; but $\{1, 3\}$ is not $(1, 2)-m_X-\beta$ -closed. Moreover, $\{2\}$, $\{1, 2\}$, and $\{2, 3\}$ is not a subset of Y and $(1, 2)-m_X-\beta$ -closed, whereas, $\{2\}$, $\{1, 2\}$, and $\{2, 3\}$ is not $(1, 2)-m_X-\beta$ -closed.

Lemma 4.11. Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) . If $m_X^{ij}Bdr_{\mathscr{B}}(A) \subseteq Y$, then $m_X^{ij}Bdr_{\mathscr{B}}(A)$ is $(i, j)-m_Y-\beta$ -closed.

Proof. It is clearly by Lemma 4.4 (1) and Lemma 4.9.

Corollary 4.12. Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) and A a subset of Y. If $m_X^{ij}Bdr_{\mathscr{B}}(A)$ is a subset of Y. Then $m_X^{ij}Bdr_{\mathscr{B}}(A) = m_Y^{ij}Bdr_{\mathscr{B}}(A)$.

Proof. Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) and A a subset of Y. This follows $m_Y^{ij}Bdr_{\mathscr{B}}(A) = Y \cap m_X^{ij}Bdr_{\mathscr{B}}(A) = m_X^{ij}Bdr_{\mathscr{B}}(A)$. The proof is completed.

Acknowledgements

We would like to thank referees for their valuable comments and suggestions. The first author is supported by Research and Academic Service Division and Faculty of Sciences and Engineering, Kasetsart University, Chalermprakiat Sakon Nakhon Province Campus.

References

- [1] C. Boonpok, Biminimal structure spaces, Int. Math. Forum. 5(15) (2010), 703-707.
- C. Boonpok, M-continuous functions on biminimal structure spaces, Far East J. Math. Sci. (FJMS) 43(1) (2010), 41-58.
- [3] C. Boonpok, C. Chokchai, M. Thongmoon and N. Viriyapong, On M^(i, j) continuous functions in biminimal space structure spaces, Int. J. Math Math. Sci. 2013 (2013), ID 381068.
- [4] J. C. Kelly, Bitopological spaces, Pro. London Math. Soc. 3(13) (1969), 71-79.
- [5] H. Maki, K. C. Rao and A. N. Gani, On generalized semi-open and preopen sets, Pure Appl. Math. Sci. 49 (1999), 17-29.
- [6] B. Mendelson, Introduction to Topology, Blackie and Son Ltd, London, 1963.
- [7] T. Noiri, The further unified theory for modifications of g-closed sets, Rendiconti del circolo Mathematico di Palermo 57(3) (2008), 411-421.
- [8] T. Noiri and V. Popa, A generalization of some forms of g-irresolute functions, Eur. J. Pure Appl. Math. 2(4) (2009), 473-493.
- [9] T. Noiri and V. Popa, On upper and lower *m*-continuous multifunctions, FILOMAT 14 (2000), 73-86.
- [10] V. Popa and T. Noiri, On *M*-continuous functions, Anal. Univ. "Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor. (2) 18(23) (2000), 31-41.





Far East Journal of Mathematical Sciences

Scimago Journal & Country Rank Enter Journal Title, ISSN or Publisher Name

Home

Journal Rankings

Country Rankings

Viz Tools

Help About Us

Far East Journal of Mathematical Sciences

Country	India	10	
Subject Area	Mathematics	IU	
Subject Category	Mathematics (miscellaneous)		
Publisher	Pushpa Publishing House	H Index	
Publication type	Journals		
ISSN	09720871		
Coverage	2008-ongoing		•••

Quartiles

Total Cites



+

http://www.scimagojr.com/journalsearch.php?q=17900156722&tip=sid&clean=0

Self-Cites